647510 **BSCabsCC6020** Seat No :________

B.Sc. Semester - 6 (*CBCS*) Examination
March/April-2023 (OLD COURSE)
MATHEMATICAL ANALYSIS-2 AND ABSTRACT ALGEBRA -2(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer the following question. (Any One) (07)

1. State and prove Heine Borel theorem.
2. Prove that a subset of a compact metric space is compact if and only if it is closed.

Q.1(B) Answer the following questions. (Any One) (04)

1. Define separated sets and justify "two disjoint sets are separated".
2. Define countable set and show that the set of all even natural numbers is countable.

Q.1(C) Answer the following question. (Any One) (03)

1. Define Open cover and compact set.
2. Define Homeomorphism and connected set.

Q.2(A) Answer the following question. (Any One) (07)

1. State and prove first shifting theorem for Laplace transform and using it evaluate $L\{e^{3t}\sin^2 4t\}$.
2. Evaluate: $L^{-1}\left\{\frac{3s+1}{(s+1)(s^2+2)}\right\}$.

Q.2(B) Answer the following questions. (Any One) (04)

1. Evaluate: $L^{-1}\left\{\frac{s}{(s+2)^2}\right\}$.
2. Evaluate: $L\{\sinh 2t \sin 3t\}$.

Q.2(C) Answer the following question. (Any One) (03)

1. Evaluate $L\{f(t)\}$, where $f(t) = \begin{cases} e^t, & 0 < t < 1 \\ 0, & t \geq 1 \end{cases}$.
2. Prove linearity of inverse Laplace transform.

Q.3(A) Answer the following question. (Any One) (07)

1. Prove that if $L\{f(t)\} = \overline{f(s)}$ and $\lim\limits_{t\to 0}\frac{f(t)}{t}$ exist then $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \overline{f(s)}\,ds$ and using that evaluate $L\left\{\frac{1-e^{2t}}{t}\right\}$.
2. State convolution theorem and using it find $L^{-1}\left\{\frac{s}{(S^2+1)(S^2+4)}\right\}$.

Q.3(B) Answer the following questions. (Any One) (04)

1. Evaluate: $L^{-1}\left(\frac{1}{s\,(s^2-2s+5)}\right)$.
2. Evaluate: $L^{-1}\left(\frac{s^2}{(s^2+9)\,(s^2+16)}\right)$.

Q.3(C) Answer the following question. (Any One) **(03)**

1. Evaluate: $L^{-1}\left(\log\left(1+\frac{a^2}{s^2}\right)\right)$.
2. Use Laplace transform to solve $y' - y = e^t, y(0) = 1$.

Q.4(A) Answer the following question. (Any One) **(07)**

1. State and prove first fundamental theorem of group homomorphism.
2. Write an example of a right ideal of a ring which not a left ideal and prove your claim.

Q.4(B) Answer the following questions. (Any One) **(04)**

1. State and prove factor theorem for polynomials.
2. Let R be a ring with unity *'e'*. Show that any proper ideal of 'R' does not contain *'e'*.

Q.4(C) Answer the following question. (Any One) **(03)**

1. Prove that kernel of a group homomorphism a subgroup of the domain.
2. Define: zero divisors, ideal.

Q.5(A) Answer the following question. (Any One) **(07)**

1. State and prove division algorithm for polynomials.
2. Show that the polynomial $f(x) = x^4 + 3x^2 - 7x + 1 \in Q[x]$ is irreducible over Q.

Q.5(B) Answer the following questions. (Any One) **(04)**

1. Factorize $f(x) = x^4 + x^3 - 2 \in Z_7[x]$ using factor theorem.
2. Simplify $[(2i + j - 3)(4i - 2j + k)^{-1}]$.

Q.5(C) Answer the following question. (Any One) **(03)**

1. Find GCD of $x^2 + 7x + 6$ and $x^2 - 5x - 6$.
2. Show that $F[x]$ is an integral domain if F is an integral domain.

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